Magneto-mechanical simulations by a coupled fast multipole method–finite element method

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Abstract

Three-dimensional magnetostatic-mechanical problems featuring infinite domains, non-linear ferromagnetic materials and large relative motions among different components are analysed by means of a numerical approach coupling a fast multipole method with both standard and non-standard finite element methods.

Keywords: Fast multipole method; BEM; FEM; Discontinuous Galerkin method

1. Introduction

The classical boundary element method (BEM), both in the collocation and the variational versions, has essentially failed to live up to initial high expectations for some reasons which can be briefly summarized as follows. (i) The BEM produces fully populated matrix equations which make the application of direct solvers unrealistic for large scale problems; moreover the computation of such matrices is numerically costly and hence iterative solvers cannot be adapted to the method as is. (ii) The BEM works appropriately for linear (and possibly isotropic) problems; non-linear problems (both from a constitutive and a geometrical point of view) can be addressed only at the cost of introducing domain integrals which spoil the boundary only nature of BEMs and represent a costly overhead in the analysis.

With respect to point (i), recent investigations of the so called fast multipole methods (FMM) seem to be changing the situation drastically. Indeed these methods, adapted to the BEM, allow to utilize iterative solvers (e.g. GMRES) and reduce the operation count per iteration to approximately $O(N \log^{\alpha} N)$ (to be compared with the $O(N^2)$ count of the classical approach), where α is a positive number. The FMM can be considered as an efficient tool for evaluating the contribution to the integral equations stemming from regions which are far apart form each other, the "near field" contributions being evaluated by means of classical tools. FMM was initially introduced by Rokhlin [9] as a fast technique for solving integral equation numerically, and later developed by Greengard as a fast solver for multibody (particle) interactions. In recent times one can observe a flourishing of different contributions and techniques ranging from the "panel clustering" method (e.g. [5]), to the "wavelet methods" (e.g. [6]) and finally to more efficient versions of the FMM (e.g. [1,3,7]).

As far as point (ii) is concerned, in many cases the optimal choice seems to be a coupled BEM–FEM approach (see e.g. [8,10]). The finite element method has obviously a dominant status in the field of computational methods in engineering, mostly because of its greater flexibility and wider range of applicability; on the contrary integral equations are superior for certain classes of problems featuring, in most cases, linear-elastic material behaviour, moving boundaries, infinite domains. A model problem which clearly evidences all the potential advantages of such coupling is depicted in Fig. 1. A vertical conductor filament A passes through a switch mechanism made of a fixed lamina B and a pivoting one C (hinged on B). A sudden increase of the current intensity through A induces an attracting force between B and C (the elastic spring is calibrated to prevent motion for usual current intensities) which eventually induces a rotation of C. This mechanism actually constitutes a part of an industrial relay.

Laminae B and C are made of a ferromagnetic highly non-linear material and are hence discretized by means of domain methods. The magnetic field in the air, on the contrary, is accounted for by the BEM, which essentially yields the following considerable advantages: (a) the infinite domain around the relay need not be discretized; (b) large rotations of the pivoting lamina C can be easily accounted for, since no remeshing is required.

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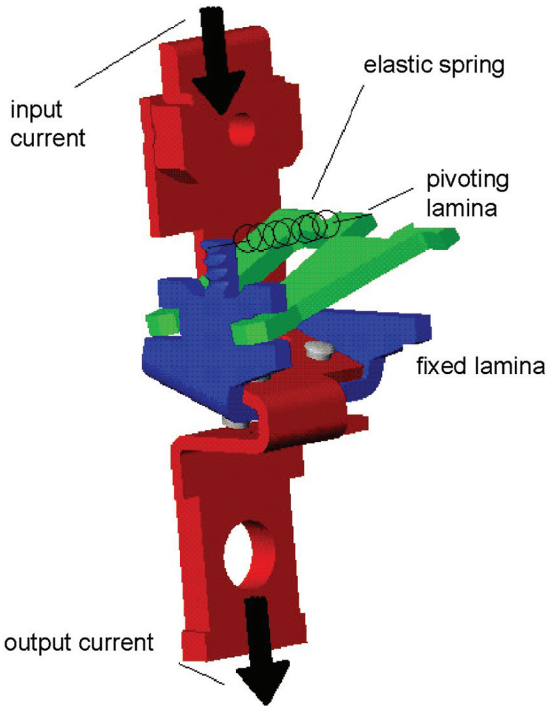

Fig. 1. Example of a relay circuit.

It is worth stressing that similar remarks also hold for several different structures, like the one in Fig. 2; a small portion of a micro-electro-mechanical-system (MEMS) is presented, which basically consists of a micro interdigited-combs capacitor.

On the basis of previous remarks, in the present paper the analysis of the structure in Fig. 1 is addressed by developing and implementing a coupled FMM–FEM as detailed in the following section.

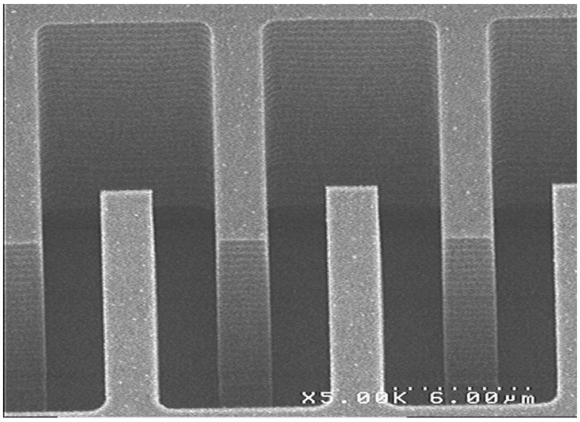

Fig. 2. Example of a micro-electro-mechanical actuator.

2. Problem formulation

Assuming that the current variation inside the conductor is slow enough to allow the use of a magnetostatic formulation, and that the distribution of the current density **J** is known, the differential equations to be solved are as follows:

$$\operatorname{curl}\mathbf{H}_f = \mathbf{J}, \qquad \operatorname{div}\mathbf{B}_f = 0 \qquad \text{in } \Omega_f \tag{1}$$

$$\operatorname{curl}\mathbf{H}_a = \mathbf{0}, \qquad \operatorname{div}\mathbf{B}_a = 0 \qquad \text{in } \Omega_a \tag{2}$$

$$(\mathbf{H}_1 - \mathbf{H}_2)\wedge\mathbf{n} = \mathbf{0}, \qquad (\mathbf{B}_1 - \mathbf{B}_2)\cdot\mathbf{n} = 0 \qquad \text{on } \partial\Omega \tag{3}$$

where **H** is the magnetic field intensity and **B** is the magnetic flux density; Ω_f represents the conductor and the ferromagnetic laminae, Ω_a the infinite domain surrounding the relay, $\partial\Omega$ is the interface between the two domains. The field quantities satisfy the constitutive relations:

$$\mathbf{B} = \mu(\mathbf{H})\mathbf{H} \quad \text{in } \Omega_f, \qquad \mathbf{B} = \mu_0\mathbf{H} \quad \text{in } \Omega_a$$

2.1. Conductor and ferromagnetic laminae

The numerical analysis of 3D magnetostatic problems with the aid of finite elements is generally addressed by means of either scalar and vector potentials, the latter option being preferred herein.

Since $\mathbf{B}_1$ is divergence free we can find $\mathbf{A}_1$ such that:

$$\mathbf{B}_1 = \operatorname{curl}\mathbf{A}_1$$

In order to completely specify the vector potential $\mathbf{A}_1$ the so called Coulomb gauge is often adopted, imposing $\operatorname{div}\mathbf{A}_1 = 0$. Eventually:

$$\operatorname{curl}\left(\frac{1}{\mu}\operatorname{curl}\mathbf{A}_1\right) = \mathbf{J} \qquad \operatorname{div}\mathbf{A}_1 = 0 \tag{4}$$

which can be recast into the equivalent variational formulation: find $\mathbf{A}_1$ and p

$$\mathbf{A}_1 \in H_0(\operatorname{curl},\Omega_f), \quad p \in H^1(\Omega_f)$$

such that

$$\int_{\Omega_f} \operatorname{curl}\tilde{\mathbf{A}}\cdot\frac{1}{\mu}\operatorname{curl}\mathbf{A}_1\,\mathrm{d}\Omega + \int_{\Omega_f}\tilde{\mathbf{A}}\cdot\operatorname{grad}p\,\mathrm{d}\Omega$$
$$= \int_{\partial\Omega}\tilde{\mathbf{A}}\cdot(\mathbf{H}_1\wedge\mathbf{n})\,\mathrm{d}\Gamma + \int_{\Omega_f}\tilde{\mathbf{A}}\cdot\mathbf{J}\,\mathrm{d}\Omega \quad \forall\tilde{\mathbf{A}}\in H_0(\operatorname{curl},\Omega_f)$$

$$\int_{\Omega_f}\operatorname{grad}\tilde{p}\cdot\mathbf{A}\,\mathrm{d}\Omega = 0 \qquad \forall\tilde{p}\in H^1(\Omega_f) \tag{5}$$

Non conventional “edge elements” are adopted in Eq. (5), since they guarantee inter-element continuity of the tangential component only, as required by the $\mathbf{A}\in H_0(\operatorname{curl},\Omega_f)$ condition. The application of discontinuous Galerkin schemes is currently under consideration.

2.2. Magnetic field in Ω_a

Applying the same procedure to Ω_a, one obtains:

$$\frac{1}{\mu_0}\operatorname{curl}\operatorname{curl}\mathbf{A}_2=\mathbf{0},\qquad \operatorname{div}\mathbf{A}_2=0$$

$$\Downarrow$$

$$\operatorname{div}\operatorname{grad}\mathbf{A}_2=\mathbf{0},\qquad \operatorname{div}\mathbf{A}_2=0 \tag{6}$$

The boundary condition $\mathbf{B}_1\cdot\mathbf{n}=\mathbf{B}_2\cdot\mathbf{n}$ implies that the tangential derivatives of $\mathbf{A}_i$ is continuous across $\partial\Omega$. Hence $\mathbf{A}_1$ and $\mathbf{A}_2$ on $\partial\Omega$ can only differ by an arbitrary constant which will be henceforth set to zero.

For a smooth point on $\partial\Omega$:

$$\frac{1}{2}\mathbf{A}(\mathbf{y})=\int_\Gamma\left[\operatorname{grad}\mathbf{A}(\mathbf{x})\cdot\mathbf{n}(\mathbf{x})\right]G(\mathbf{y},\mathbf{x}) - \mathbf{A}(\mathbf{x})\left[\operatorname{grad}G(\mathbf{y},\mathbf{x})\cdot\mathbf{n}(\mathbf{x})\right]\,\mathrm{d}\Gamma \tag{7}$$

where kernel $G(\mathbf{x},\mathbf{y})$ is the classical potential theory Kelvin kernel and reads:

$$G=\frac{1}{4\pi}\frac{1}{r}$$

and the condition $\operatorname{div}\mathbf{A}_2=0$ is automatically satisfied. Eq. 7 can be transformed into:

$$\frac{1}{2}\mathbf{A}_2=\int_\Gamma\Big(\left[\mu_0\mathbf{H}_2\wedge\mathbf{n}\right]G-\left[\operatorname{grad}G\cdot\mathbf{n}\right]\mathbf{A}_2 + \left[\operatorname{grad}G\wedge\mathbf{n}\right]\mathbf{A}_2\Big)\,\mathrm{d}\Gamma \tag{8}$$

which is more suitable for coupling with Eq. (5). Eq. (8) is collocated at every node of the interface $\partial\Omega$ or enforced in a variational form.

The continuity relations:

$$\mu\mathbf{H}_1\wedge\mathbf{n}=\mu_0\mathbf{H}_2\wedge\mathbf{n},\qquad \mathbf{A}_1=\mathbf{A}_2\qquad \text{on }\partial\Omega$$

complete the set of governing equations.

3. Future perspectives

The formulation briefly described in the previous section is being implemented employing fast multipole techniques for the acceleration of Eq. (8), edge elements for Eq. (5), and accounting for the highly non-linear constitutive behaviour of the ferromagnetic materials through an iterative and auto-adaptive procedure. Numerical results will be provided and commented extensively.

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